Exercise 95

(a) If n is a positive integer, prove that

$$\frac{d}{dx}(\sin^n x \cos nx) = n \sin^{n-1} x \cos(n+1)x$$

(b) Find a formula for the derivative of $y = \cos^n x \cos nx$ that is similar to the one in part (a).

Solution

Part (a)

Take the derivative of $\sin^n x \cos nx$.

$$\begin{aligned}
\frac{d}{dx}(\sin^n x \cos nx) &= \left[\frac{d}{dx}(\sin^n x)\right] \cos nx + \sin^n x \left[\frac{d}{dx}(\cos nx)\right] \\
&= \left[(n \sin^{n-1} x) \cdot \frac{d}{dx}(\sin x)\right] \cos nx + \sin^n x \left[(-\sin nx) \cdot \frac{d}{dx}(nx)\right] \\
&= \left[(n \sin^{n-1} x) \cdot (\cos x)\right] \cos nx + \sin^n x \left[(-\sin nx) \cdot (n)\right] \\
&= n(\sin^{n-1} x)(\cos x \cos nx - \sin nx \sin x) \\
&= n \sin^{n-1} x \cos(x + nx) \\
&= n \sin^{n-1} x \cos(n+1)x
\end{aligned}$$

Part (b)

Take the derivative of $\cos^n x \cos nx$.

$$\begin{aligned}
\frac{d}{dx}(\cos^n x \cos nx) &= \left[\frac{d}{dx}(\cos^n x)\right] \cos nx + \cos^n x \left[\frac{d}{dx}(\cos nx)\right] \\
&= \left[(n \cos^{n-1} x) \cdot \frac{d}{dx}(\cos x)\right] \cos nx + \cos^n x \left[(-\sin nx) \cdot \frac{d}{dx}(nx)\right] \\
&= \left[(n \cos^{n-1} x) \cdot (-\sin x)\right] \cos nx + \cos^n x \left[(-\sin nx) \cdot (n)\right] \\
&= -n(\cos^{n-1} x)(\sin x \cos nx + \sin nx \cos x) \\
&= -n \cos^{n-1} x \sin(x + nx) \\
&= -n \cos^{n-1} x \sin(n+1)x
\end{aligned}$$